[Total No. of Questions - 9] [Total No. of Printed Pages - 4]

**Dec.-22-1353**

**MA-101L (Applied Mathematics-I)**

**B.Tech. 1st (CBCS)**

**Time : 3 Hours** **Max. Marks : 60**

*The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.*

**Note :** Attempt Five questions in all, selecting one question each from Section A, B, C and D. Section E is compulsory.

## SECTION - A

1. (a) Prove that the sequence:

$$\left\{\frac{2n-7}{3n+2}\right\}$$

is (i) monotonically increasing (ii) bounded and (iii) convergent. (5)

(b) Show that the alternating series:

$$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots\ldots\ldots\ldots$$ is convergent. (5)

2. (a) Find the radius of convergence of the power series:

$$\sum_{n=1}^{\infty}\frac{(n-1)!}{n^n}x^n$$ (5)

(b) Show that the function represented by:

$\sum_{n=1}^{\infty}\frac{\sin nx}{n^3}$ is differentiable for every x and its derivative is

$$\sum_{n=1}^{\infty}\frac{\cos nx}{n^2}.$$ (5)



## SECTION - B

3. (a) State and prove Canchy's Mean Value Theorem. (5)

(b) By using Lagrange's Mean Value Theorem, prove that,

$|\sin x-\sin y|\le|x-y|$ for all $x, y \in R$. (5)

4. (a) Find the area bounded by the parabola $y=2x-x^2$ and the x-axis. (5)

(b) Find the volume of the solid generated by revolving the ellipse:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

about the x-axis. (5)

## SECTION - C

5. (a) Discuss the continuity of f(x, y) at (0, 0) where

$$f(x,y)=\begin{cases}\dfrac{2xy^2}{x^3+y^3}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$ (5)

(b) Find all the maxima and minima of the function:

$f(x,y)=x^3+y^3-63(x+y)+12xy$ (5)

6. (a) If $u=\sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, show that

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\tan u$$ (5)

(b) If $x=r\cos\theta,\ y=r\sin\theta$, prove that

$\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}=0$ except when x = 0, y = 0. (5)

**[P.T.O.]**

## SECTION - D

7. (a) Evaluate :

$$\int_1^2\int_0^3 (x^2+y^2)\,dx\,dy. \qquad (5)$$

(b) Change the order of integration and evaluate the integral:

$$\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dy\,dx. \qquad (5)$$

8. (a) Evaluate:

$\iiint (x+y+z)\,dx\,dy\,dz$ over the tetrahedron bounded by the planes x=0, y=0, z=0, x+y+z=1. (5)

(b) Find the area enclosed by the parabolas $y^2=4ax$ and $x^2=4ay,\ a>0$ by using the double integration. (5)

## SECTION - E (Compulsory)

9. Attempt all the questions:

(a) Define convergent sequence.

(b) State Couchy's Second Theorem on Limits.

(c) Define absolutely convergent series.

(d) Find the value of c of the Lagrange's mean value theorem, if $f(x)=x(x-1)(x-2)$ in $\left[0,\frac{1}{2}\right]$.

(e) Define rectifiable curve.

(f) Prove that :

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}=0.$$



(g) State Euler's theorem for a homogeneous function of three variables.

(h) What are the necessary condtions for maxima and minima of f(x,y)?

(i) Find the area of the circle using the double integration.

(j) State Cauchy's Root Test. (10×2=20)