[Total No. of Questions - 9] [Total No. of Printed Pages - 4]
(2125)

15023

B. Tech 1st / 2nd Semester Examination

Applied Mathematics-I (OS)

AS-1001

Time : 3 Hours **Max. Marks : 100**

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt five questions in all, selecting one question each from Section A, B, C & D. Section E (question 9) is compulsory.

SECTION - A

1. (a) Show that the following function is discontinuous at (0,0)

$$f(x,y)=\begin{cases}\dfrac{xy^3}{(x^2+y^6)} & ; \quad (x,y)\neq(0,0)\\ 0 & ; \quad (x,y)=(0,0)\end{cases} \qquad (10)$$

(b) Using Jacobian, show that the functions $u=x^2+y^2+z^2$, $v=x+y+z$, $w=xy+yz+zx$ are not independent on one another. Hence find the relation between them. (10)

2. (a) Examine the maxima, minima and saddle points of the function

$u = x^2 + xy + 3x + 2y + 5$ (10)

(b) use Lagrange's method to determine to find the minimum value of xy when $x^2 + xy + y^2 = a^2$ (10)

[P.T.O.]

SECTION - B

3. (a) Find the length of the curve y = log(cos x) from $0 \le x \le \frac{\pi}{4}$. (10)

(b) Find the general solution of $\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}-\frac{dy}{dx}-y=\sin 2x$ (10)

4. (a) Find the general solution of $\frac{d^2y}{dx^2}+y=\sec x$; using method of variation of parameters. (10)

(b) Use Maclaurin's Theorem to show that

$$e=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}\ldots\frac{x^{n-1}}{(n-1)!}+\frac{x^n}{n!}e^{\theta s}.0<\theta<1 \qquad (10)$$

(Symbol ! stands for factorial)

SECTION - C

5. (a) For what values of λ and μ, the system of equations has

x + y + 5z = 0

x + 2y + 3λz = μ

x + 3y + λz = 1

(i) no solution (ii) unique solution (iii) more than one solution? (10)

(b) Prove that the eigen values of skew-Hernitian matrices are all pure imaginary. (10)

6. (a) Define bilinear form on vector space V. When a bilinear form is said to be symmetric and skew-symmetric? Write the corresponding bilinear form of $A=\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}$. (10)

(b) Find the eigen values of the matrix $A = \begin{bmatrix} 3 & 2-i & -3i \\ 2+i & 0 & 1-i \\ 3i & 1+i & 0 \end{bmatrix}$ (10)

SECTION - D

7. (a) Show that the function f(x)=xy+iy is everywhere continuous but not analytic. (10)

(b) Show that function f(z) = coshx coshy is harmonic and find its conjugate. (10)

8. (a) Show that an analytic function with constant modulus is constant. (10)

((b) Prove that the function $f(z) = |z|^2$ is continuous everywhere but nowhere differential except at the origin. (10)

SECTION - E

9. (a) Define skew-Hermitian matrix, with example.

(b) Verify Euler's Theorem for a function $z = \tan^{-1}\dfrac{x+y}{\sqrt{x}+\sqrt{y}}$

(c) Show that $\int_0^3\int_0^2\int_0^1 (x+y+z)dzdxdy = 18$

(d) Solve $\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} + 2y = 0$

(e) Define beta function.

(f) for what value of λ, the system $\begin{bmatrix} 1 & 2 \\ 3 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ has more than one solution?

(g) State Euler's Theorem for homogeneous function of n variables.

(h) Define eigen values and eigen vectors of a matrix.

(i) Show that Cauchy Riemann Equations are satisfied for a function

$e^x(\cos y + i \sin y)$

(j) Define analytic function. (2×10=20)