Roll No. ....

Total Pages : 05

J-FB-22-00207

B. Tech. EXAMINATION, 2022

Semester I (CBCS)

ENGINEERING MATHEMATICS–I (A & B)

MA-101

Time : 3 Hours *Maximum Marks : 60*

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt *Five* questions in all, selecting *one* question from each Sections A, B, C and D. Q. No. 9 is compulsory.

Section A

1. (a) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ 5

(b) For what values of λ and μ does the system of equations 5

$x + y + z = 6, \quad x + 2y + 3z = 10, \quad x + 2y + \lambda z = \mu$

have

(i) Unique solution

(ii) More than one solution

(iii) No solution ?

2. State Cayley-Hamilton Theorem. Verify this theorem for the matrix : 10

$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ Hence find A^{-1}

Section B

3. (a) If $\tan(\log(x+iy)) = a + ib$ and $a^2 + b^2 \neq 1$, then prove that $\tan\left(\log\left(x^2+y^2\right)\right) = \frac{2a}{1-a^2-b^2}$ 5

(b) Find the cub root of unity. 5

4. (a) Sum the series

$1 + x\cos\alpha + x^2\cos 2\alpha + x^3\cos 3\alpha + \ldots$ to n terms, where x is less than unity. Also find the sum to infinity. **5**

(b) Separate real and imaginary parts of

$\sin^{-1}(\cos\theta + i\sin\theta)$, $0 < \theta < \frac{\pi}{2}$ **5**

Section C

5. (a) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{-\sin u\cos 2u}{4\cos^3 u}$$ **5**

(b) Examine the funtion $x^3 + y^3 - 3axy$ for maxima and minima **5**

6. (a) Evaluate the integral **5**

$$\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} \frac{1}{\sqrt{1-x^2-y^2-z^2}}\,dz\,dy\,dx$$

(b) Evaluate $\iint_R xy\,dx\,dy$, where R is the region bounded by the parabolas **5**

$y^2 = 4x$ and $x^2 = 4y$.

Section D

7. (a) Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r}f'(r)$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. **5**

(b) Find the directional derivative of the function $2xy + z^2$ at the point $(1,-1,3)$ in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$. **5**

8. (a) Use Gauss Divergence theorem for $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ taken over the region bounded by the cylinder $x^2 + y^2 = 4$, $z = 0$, $z = 3$. **5**

(b) Use Green's theorem to evaluate $\oint_C \left((2x^2 - y^2)dx + (x^2 - y^2)dy\right)$, where C is the boundary in xy-plane of area enclosed by the x-axis and semicircle $x^2 + y^2 = 1$ in the upper half xy-plane. **5**

(Compulsory Question)

9. Attempt all the ten parts . $2 \times 10 = 20$

(a) If A is an orthogonal matrix, prove that $|A| = \pm 1$.

(b) Separate $\sin(x+iy)$ into real and imaginary parts.

(c) If λ be an eigen value of a non-singular matrix A, then show that λ^{-1} is an eigen value of A^{-1}.

(d) Prove that e^z is a periodic function, where z is a complex function.

(e) Prove the $\log i^i = -\left(2n + \frac{1}{2}\right)\pi$, where $i = \sqrt{-1}$ and $n = 0, 1, 2, 3......$

(f) Find first order partial derivative of $u = \cos^{-1}\left(\frac{x}{y}\right)$.

(g) Prove that $\beta(m,n) = \beta(m+1,n) + \beta(m,n+1)$; $m > 0$, $n > 0$ and β is Beta function.

(h) $\int_0^1 \int_0^1 e^{x+y} \, dy \, dx$.

(i) State complex matrix and give *one* example.

(j) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then prove that $\text{curl}\,\vec{r} = 0$