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(2067)

1702(J)

B. Tech 1st Semester Examination

Engineering Mathematics-I (CBS)

MA-101

Time : 3 Hours **Max. Marks : 60**

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Candidate is required to attempt five questions in all, selecting one question each unit and question no. 9 is compulsory.

UNIT - I

1. (a) Define linear dependent and linear independent vectors. Are the vectors $x_1 = (1, 3, 4, 2)$, $x_2 = (3, -5, 2, 2)$ and $x_3 = (2, -1, 3, 2)$ linearly dependent? If so express one of these as a linear combination of the others. (6)

   (b) Investigate the values of λ and μ so that the equations

   $2x + 3y + 5z = 9$, $7x + 3y - 2z = 8$, $2x + 3y + \lambda z = \mu$, have (i) no solution, (ii) a unique solution and (iii) an infinite number of solutions. (6)

2. (a) Show that the eigen values of a unitary matrix have the absolute value 1. (6)

   (b) Find the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$, and hence find its inverse. (6)

UNIT - II

3. (a) Prove that the n^{th} roots of unity form a geometric progression. Also show that the sum of these n roots is zero and their product is $(-1)^{n-1}$. (6)

   (b) Sum the series by C+iS method

   $$\sin^2\theta - \frac{1}{2}\sin 2\theta \sin^2\theta + \frac{1}{3}\sin 3\theta \sin^3\theta - \frac{1}{4}\sin 4\theta \sin^4\theta + \ldots\ldots\infty.$$

   (6)

4. (a) If f(z) be an analytic function of z, show that

   $$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|Rf(z)|^2 = 2|f'(z)|^2.$$ (6)

   (b) Find the analytic function, whose real part is $\frac{\sin 2x}{(\cosh 2y - \cos 2x)}$. (6)

UNIT - III

5. (a) If $u = \left[\frac{x^{1/3} + y^{1/3}}{x^{1/2} + y^{1/2}}\right]^{1/2}$, then prove that

   $$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -\frac{1}{12}u.$$ (6)

   (b) A rectangular box open at the top is to have volume of 32 cubic ft. Find the dimension of the box requiring least material for its constructions by using Lagrange's Method of undetermined multipliers. (6)

6. (a) Change the order of integration in $1 = \int_0^1 \int_{x^2}^{2-x} xy \, dx \, dy$, and hence evaluate the same. (6)

(b) Find the volume (by triple integrals) of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ (6)

UNIT - IV

7. (a) Find the directional derivatives of $f(x, y, z) = xy^2 + yz^3$ at the point (2, −1, 1) in the direction of vector $\hat{i}+2\hat{j}+2\hat{K}$ (6)

(b) Discuss physical interpretation of divergence. (6)

8. (a) If velocity vector is $F = y\hat{i}+2\hat{j}+xz\hat{K}$ m/sec, show that the flux of water through the parabolic cylinder $y = x^2$, $0\le x\le 3$, $0\le z\le 2$ is $69m^3$/sec. (6)

(b) If S is any closed surface enclosing a volume V and $F = ax\hat{i}+by\hat{j}+cz\hat{K}$, prove by using Gauss Divergence theorem that $\int_S F.\hat{N}ds=(a+b+c)V$. (6)

UNIT - V (Compulsory)

9. Each part of this question carry one mark.

(a) Define characteristic matrix and characteristic equation of a matrix.

(b) Define orthogonal matrix and unitary matrix.

(c) Prove that logarithm of a complex number is a many-valued function.

(d) Define limit of a complex function f(z).

(e) If $u=f\left(\frac{y}{x}\right)$, then show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$.

(f) Write down the necessary and sufficient conditions for extrema of a function f of two variables.

(g) Evaluate $\int_0^5\int_0^{x^2} x(x^2+y^2)dxdy$.

(h) Change the order of integration in the integral $I=\int_{-a}^{a}\int_0^{\sqrt{(a^2-y^2)}} f(x,y)dxdy$.

(i) Any motion in which the curl of the velocity vector is zero is said to be ....

(j) If $R = x\hat{i}+y\hat{j}+z\hat{K}$, show that $\nabla\times R=0$.

(k) State Green's theorem.

(l) Using divergence theorem, prove that $\int_s \nabla r^2.dS=6V$.

(12×1=12)