

Dec.-22-1355

CS-102L (Mathematical Concepts for Applied Intelligence)

B.Tech. 1st (CBCS)

Time : 3 Hours

Max. Marks : 60

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt Five questions in all, selecting one question each from sections A, B, C and D. Question no. 9 is compulsory.

SECTION - A

1. (a) Solve the equation:

$$7x - \frac{3}{4} = \frac{1}{5} - x \quad (5)$$

- (b) If α and β are the zeros of the quadratic polynomial $f(x) = kx^2 + 4x + 4$ such that $\alpha^2 + \beta^2 = 24$, Find the values of k . (5)

2. (a) Determine two positive numbers whose sum is 300 and whose product is maximum. (5)

- (b) Find a, b, c so that the function :

$$f(x) = \begin{cases} x^3 + 3x + 5, & x \leq 1 \\ ax^2 + bx + c, & x > 1 \end{cases}$$

is twice differentiable at $x = 1$. (5)

SECTION - B

3. (a) For what value of λ the vectors:

$$\hat{i} + 2\hat{j} + 3\hat{k}, \lambda\hat{i} - \hat{j} - \hat{k} \text{ and } 3\hat{i} - 4\hat{j} + 3\hat{k} \text{ are coplanar? (5)}$$

- (b) Find a unit vector coplanar with $\hat{i} + \hat{j} + 2\hat{k}$, $\hat{i} + 2\hat{j} + \hat{k}$ and perpendicular to $\hat{i} + \hat{j} + \hat{k}$. (5)

4. (a) Solve the following system of linear equations by the matrix method:

$$2x + y - z = 6$$

$$3x - y + 2z = 3$$

$$x + 2y - z = 5 \quad (5)$$

- (b) Find all the eigen values and eigen vectors of the following matrix:

$$\begin{bmatrix} 4 & 2 & -2 \\ 2 & 5 & 0 \\ -2 & 0 & 3 \end{bmatrix} \quad (5)$$

SECTION - C

5. (a) If F is the distributive function of the random variable X and if $a < b$, then prove that

$$P(a < X \leq b) = F(b) - F(a) \quad (5)$$

- (b) State and Prove Baye's Theorem. (5)

6. (a) A random variable X has exponential distribution with parameter $\lambda = 5$. Find its p.d.f., mean, variance and moment generating function. (5)

- (b) Find the probability distribution of the number of heads when three coins are tossed simultaneously. (5)

SECTION - D

7. (a) Describe Merits and Limitations of sampling. Also write a note on systematic sampling. (5)

[P.T.O.]

- (b) Describe briefly the procedure of Testing Hypothesis. (5)
8. (a) Test the significance of the correlation $r = 0.5$ from a sample of size 18 against hypothesis correlation $\rho = 0-0.7$. (5)
- (b) (i) Write the properties of a Good Estimator.
- (ii) A coin was tossed 400 times and the head turned up 216 times. Test the hypothesis that the coin is unbiased. (5)

(Compulsory Question)

9. Answer all the questions :

- (a) Find the monic quadratic equation with 3 and 5 as its roots.
- (b) Simplify:

$$\sqrt[4]{81x^8y^4}$$
- (c) Show that the following vectors are coplanar:
 $\bar{a} = 10\hat{i} - 12\hat{j} - 4\hat{k}$, $\bar{b} = -16\hat{i} + 22\hat{j} - 2\hat{k}$ and
 $\bar{c} = 2\hat{i} - 8\hat{j} + 16\hat{k}$
- (d) Find the extreme values of :

$$f(x) = x^4 - 2x^2 + 3$$
- (e) If A is an n-rowed non-singular matrix, then prove that
 $f(A^{-1}) = f(A)$.
- (f) Find the characteristic equation and eigen values of the following matrix:

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

- (g) Three coins are tossed once find the probability that at least one head and one tail occur.
- (h) Define Moment Generating Function (m.g.f).
- (i) Define Central Limit Theorem.
- (j) What do you mean by classification of data? (10×2=20)