[Total No. of Questions - 9] [Total No. of Printed Pages - 4]
(2066)

16024(J) J-16

B. Tech 2nd Semester Examination

Engineering Mathematics-II (NS)

NS-104

Time : 3 Hours **Max. Marks : 100**

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt five questions in all, selecting one question from each section A, B, C & D of the question paper and all the subparts of the question in section E. Use of non-programmable calculator is allowed.

SECTION - A

1. (a) Determine the nature of the following series

$$\frac{1}{x}+\frac{2!}{x(x+1)}+\frac{3!}{x(x+1)(x+2)}+\ldots(x>0)$$

(b) Discuss the convergence of the series

$$\frac{2}{1^p}+\frac{3}{2^p}+\frac{4}{3^p}+\frac{5}{4^p}+\ldots\infty$$ (10+10+20)

2. (a) Examine the behavior of the series $\sum_{n=1}^{\infty}\frac{(n+1)^n x^n}{n^{n+1}}$

(b) Test the following series for convergence and absolute convergence

$$1+\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}+\frac{1}{4\sqrt{4}}+\ldots\infty$$ (10+10=20)

[P.T.O.]

SECTION - B

3. (a) Find a Fourier series of the function f(x) = |sin x| in the interval $(-\pi, \pi)$

(b) Obtain half-range cosine series of

$$f(x)=\begin{cases}kx & 0\le x\le \frac{l}{2}\\ k(l-x) & \frac{l}{2}\le x\le l\end{cases}$$

Hence deduce that $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\infty$

(10+10=20)

4. (a) Find the Fourier series of $f(x) = x^2$ in $-\pi<x<\pi$. Use Parseval's identity to prove that $\frac{\pi^2}{90}=1+\frac{1}{2^4}+\frac{1}{3^4}+\ldots$

(b) Obtain the Fourier Sine series for f(x) = 1 in $0<x<\pi$ and hence show that

$$\frac{\pi^2}{8}=1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\ldots.$$ (10+10=20)

SECTION - C

5. (a) Solve (cosx + ysinx)dx–(cosx)dy=0, $y(\pi)=0$

(b) Using the method of variation of parameter solve

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=e^x\tan x$$ (10+10=20)

6. (a) Solve $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$

(b) Solve $xyp^2 + (3x^2 - 2y^2)p - 6xy = 0$, where $p = \frac{dy}{dx}$ (10+10=20)

SECTION - D

7. (a) Find the curvature and torsion of the curve $x = a\cos t$, $y = a\sin t, z = bt$.

(b) State Green's Theorem and evaluate $\int_C (2xy - x^2)dx + (x^2 + y^2)dy$ where C is the boundary of the region enclosed by $y = x^2$ and $y^2 = x$. (10+10=20)

8. (a) If S is any closed surface enclosing a volume V and $\vec{F} = x\hat{i} + 2y\hat{j} + 3z\hat{k}$, prove that $\iint_S \vec{F}.\hat{n}dS = 6V$.

(b) Evaluate $\iint_S \vec{A}.\hat{n}dS$, where $\vec{A} = z\hat{i} + x\hat{j} - 3y^2z\hat{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$. (10+10=20)

SECTION - E

9. (i) If $\vec{F}(t)$ has a constant magnitude then $\vec{F}.\frac{\partial \vec{F}}{\partial t} = 0$.

(ii) Show that $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla.\vec{A}) - \nabla^2\vec{A}$

(iii) Test whether the following series is absolutely convergent or conditionally convergent?

$$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots \infty$$

(iv) Solve $(y - px)(p - 1) = p$ where $p = \frac{dy}{dx}$.

[P.T.O.]

(v) Show that $\frac{1}{D+a}X = e^{-ax}\int Xe^{ax}dx$ where $D = \frac{d}{dx}$.

(vi) Find the particular integral of $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = e^x$.

(vii) State Parseval's formula.

(viii) Define positive term series and alternating series with example.

(ix) Discuss Fourier series for even and odd function.

(x) Give the physical significance of Curl. (10×2=20)