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B. Tech 3rd Semester Examination

Engineering Mathematics-III (NS)

NS-206

Time : 3 Hours Max. Marks : 100

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Candidates are required to attempt five questions in all selecting one question from each of the section A, B, C and D of the question paper and all the sub parts of the question in section E. Use of non-programmable calculators is allowed.

SECTION - A

1. (a) Form the partial differential equation from $\phi(x^2+y^2+z^2, xyz)=0$.

(b) Solve the partial differential equation pxy + pq + qy = yz by Charpit's method.

(c) Solve $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial x \partial y}-6\frac{\partial^2 z}{\partial y^2}=y\cos x$ (6+7+7=20)

2. (a) The points of trisection of a string are pulled aside through the same distance on opposite sides of the position of equilibrium and the string is released from rest. Derive an expression for the displacement of the string at subsequent time and show that the mid-point of the string always remains at rest.

(b) Solve the equation $\frac{\partial u}{\partial x}=2\frac{\partial u}{\partial t}+u$, given that $u(x,0)=6e^{-3x}$ by using the method of separation of variables. (15+5)

SECTION - B

3. (a) Find the complete solution of the differential equation

$$(1-x^2)y''-2xy'+n(n+1)y=0$$

when n is a real number.

(b) If α and β are the roots of $J_n(x) = 0$, then prove that

$$\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx=\begin{cases}0, & \text{when } \alpha\neq\beta\\ \frac{1}{2}J_{n+1}^2(x), & \text{when } \alpha=\beta\end{cases}$$ (10+10)

4. (a) Discuss Frobenius method when x = 0 is a regular singular point of the differential equation

$$P_0(x)\frac{d^2y}{dx^2}+P_1(x)\frac{dy}{dx}+P_2(x)y=0.$$

(b) Prove that $J_{n+1}(x)=\frac{2n}{x}J_n(x)-J_{n-1}(x)$ and hence show that

$$J_{\frac{5}{2}}(x)=\sqrt{\frac{2}{\pi x}}\left(\frac{3-x^2}{x^2}\sin x-\frac{3}{x}\cos x\right)$$ (10+10=20)

SECTION - C

5. (a) Solve the IVP by the Laplace Transform method

$$\frac{dy}{dx}+2y+\int_0^t y\,dt=\sin t,\quad y(0)=1.$$

(b) (i) Find Laplace transform of $\int_0^\infty t^2e^{3t}\sin^2 t\,dt$

(ii) Find the inverse Laplace transform of $\frac{1}{9s^2+6s+1}$

(10+10=20)

[P.T.O.]

6. (a) Find the Fourier transform of $f(x)=\begin{cases}1-x^2 & \text{if } |x|\le 1\\ 0 & \text{if } |x|>1\end{cases}$ and use it to evaluate $\int_0^{\infty}\left(\frac{x\cos x-\sin x}{x^3}\right)\cos\left(\frac{x}{2}\right)dx$.

(b) State and prove Convolution theorem for complex Fourier Transform. (10+10=20)

SECTION - D

7. (a) Use Cauchy's integral formula to evaluate

$\int_C \frac{4-3z}{z(z-1)(z-2)}dz$, where C is the circle $|z|=\frac{3}{2}$.

(b) Show that the function $u=\frac{1}{2}\log(x^2+y^2)$ is harmonic. Find its harmonic conjugate. (10+10=20)

8. (a) Expand $f(z)=\frac{1}{(z+1)(z+3)}$, for $1<|z|<3$ by Laurent's series expansion.

(b) Evaluate the following integral using Residue theorem

$\int_C \frac{z^2}{(z-1)^2(z+2)}dz$, where $C:|z|=3$. (10+10=20)

SECTION - E

9. (a) Solve the partial differential equation:

$x(z^2-y^2)p+y(x^2-z^2)q=z(y^2-x^2)$

(b) Write two-dimensional heat equation. Also obtain heat equation in the steady state.

(c) State first shifting theorem of Fourier transforms.

(d) Find the Laplace transform of error function.

(e) Define Unit Step function.

(f) Define removable singularity. Explain with an example.

(g) Write the C-R equations in polar form.

(h) Give the importance of Laurent's series.

(i) Explain the method of separation of variable to solve partial differential equation.

(j) Define ordinary, singular and regular singular point of the differential equation. (10×2=20)