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**B. Tech 4th Semester Examination**  
**Probability/Statistics/Queuing Theory (OS)**

AS-4001

Time : 3 Hours

Max. Marks : 100

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

**Note :** Attempt five questions in all, selecting one question from each section A, B, C and D and section E is compulsory. Use of non programmable calculators is allowed.

**SECTION - A**

1. (a) Find  $k$  such that  $f(x)$  is a probability density function of a continuous random variable  $X$ , where  $f(x)$  is defined as follows

$$f(x) = \begin{cases} ke^{-3x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

Also find  $P(0.5 \leq x \leq 1)$ .

- (b) Three machines A, B, C produce respectively 60%, 30% and 10% of the total number of items of a factory. The percentage of respective outputs of these machines is 2%, 3% and 4%, respectively. An item is selected at random and is found to be defective. Find the probability that the item was produced by machine C.
- (c) A is one of the 6 horses entered for a race and is to be ridden by one of the 2 jockeys B and C. It is 2 to 1 that B rides A, in which case all the horses are equally likely to win, with rider C, A's chances tripled, (i) Find the probability that A wins. (ii) What are odds against A's winning? (7+7+6=20)

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2. (a) A random variable  $X$  has the following probability distributions

$x_i$	0	1	2	3	4
$P(X=x_i)$	3k	3k	k	2k	6k

Find (i)  $k$  (ii) Mean (iii)  $P(X > 2)$ .

- (b) A and B throw alternately with a pair of dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A begins, find his chance of winning.
- (c) Five defective bulbs are accidentally mixed with twenty good ones. It is not possible to just look at a bulb and tell whether or not it is defective. Find the probability distribution of the number of defective bulbs, if four bulbs are drawn at random from this lot. (7+6+7=20)

**SECTION - B**

3. (a) Assume that 50% of all engineering students are good in Mathematics; determine the probability that among 18 engineering students (i) exactly 10, (ii) at least 10. (iii) at most 8, (iv) at least 2 and at most 9, are good in Mathematics.
- (b) Define Moment generating function. Also find the moment generating function of Poisson distribution and hence calculate its mean and variance.
- (c) A typist types 3 letters erroneously for every 100 letter. What is the probability that the tenth letter typed is the first erroneous letter? (8+8+4=20)
4. (a) State and prove Central Limit Theorem.
- (b) In a test on 200 electric bulbs, it was found that the life of a particular make is normally distributed with an average life of 204 hours and standard deviation 6 hours. Estimate the number of bulbs likely to burn for :
- (i) More than 215 hours  
(ii) Less than 195 hours  
(iii) More than 192 hours but less than 216 hours.

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- (c) Find moment generating function of normal distribution and prove that  $\mu_{2n+1}=0$ ,  $\mu_{2n}=(2n-1)(2n-3)\dots 3.1 \sigma^{2n}$  and hence  $\beta_1=0$  and  $\beta_2=3$ . (7+6+7=20)

## SECTION - C

5. (a) Using the data set of height vs length for 10 men in the following table:

x	70.8	66.2	71.7	68.7	67.6	69.2	66.5	67.2	68.3	65.6
y	42.5	40.2	44.4	42.8	40	47.3	43.4	40.1	42.1	36

Find the correlation coefficients

- (b) For the following data, find the regression line

X	1	2	3	4	5	8	10
Y	9	8	10	12	14	16	15

(10+10=20)

6. (a) Using the data given below, find the demand when the price of the quantity is Rs. 12.50.

	Price	Demand
Means	10	35
Standard Deviation	2	5

Coefficient of correlation (r) = 0.8

- (b) The regression equations calculated from a given set of observations

$$x = -0.4y + 6.4$$

$$y = -0.6x + 4.6$$

Calculate  $\bar{x}$ ,  $\bar{y}$  and  $r_{xy}$ . (10+10=20)

## SECTION - D

7. (a) Patients arrive at a clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate of 20 per hour.

- (i) Find the effective arrival rate at a clinic.

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- (ii) What is the probability that an arriving patient will not wait?

- (iii) What is the expected waiting time until a patient is discharged from the clinic?

- (b) At a one-man barber shop, the customers arrive following Poisson process at average rate of 5 per hour and they are served according to the exponential distribution with an average service rate of 10 minutes. Assuming that only 5 seats are available for waiting customers, find the average time a customer spends in the system.

(10+10=20)

8. (a) At a telephone booth, the arrivals are on the average 15 per hour. A call on the average takes 3 minutes. There is just one phone (Poisson and exponential arrival), find:  
(i) The expected numbers of customers at the booth (ii) The idle time of the booth.

- (b) Write the characteristics of Model (M/M/1:  $\infty$  FCFS).

(10+10=20)

## SECTION - E

9. (i) State Pollaczek-Kinchin Transform Formula.  
(ii) Discuss Queuing Theory and its various models.  
(iii) Write a short note on Poisson process.  
(iv) What is the difference between correlation index and correlation ratio?  
(v) Define dispersion and also tell the measure of dispersion.  
(vi) Define correlation coefficients. Also discuss its properties.  
(vii) State central limit theorem.  
(viii) Find the four moment of the following series about mean: 3, 6, 8, 10, 18.  
(ix) Define continuous random variables.  
(x) State geometric probability. (10×2=20)